

# Direct Detection of Neutralino Dark Matter and the Anomalous Dipole Moment of the Muon

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## Abstract

We compare predictions for the spin-independent contribution to the neutralino-proton scattering cross section  $\sigma_{\chi p}$  and for the anomalous magnetic dipole moment of the muon,  $a_\mu = (g_\mu - 2)/2$ , in models with gravity-mediated supersymmetry breaking. We nearly always find a positive correlation between these two measurables, i.e. scenarios with larger  $a_\mu$  also tend to have larger  $\sigma_{\chi p}$ , but the detailed prediction differs greatly between models. In particular, we find that for the popular mSUGRA scenario with universal soft breaking masses at the scale of Grand Unification, measurements of  $a_\mu$  currently seem more promising. On the other hand, if scalar soft breaking masses at the GUT scale receive sizable contributions from  $SO(10)$  D-terms, one often finds scenarios with large  $\sigma_{\chi p}$  but  $a_\mu$  below the currently foreseen sensitivity. A string-inspired model with non-universal scalar spectrum at the GUT scale falls between these two cases.

# 1 Introduction

The Minimal Supersymmetric Standard Model (MSSM) [1] is one of the best motivated extensions of the Standard Model. It offers a natural solution of the hierarchy problem [2] as well as amazing gauge coupling unification [3]. Since naturalness requires that at least some superparticles have masses at or below the TeV scale, supersymmetric theories generally predict a rich phenomenology at future colliders [4]. As extra “bonus”, the simplest version of the MSSM, where  $R$ -parity is conserved, also contains a new stable particle (the lightest supersymmetric particle, LSP); in most cases this is the lightest neutralino, which often makes a good Dark Matter candidate [5].

Unfortunately, direct searches for superparticles at high energy colliders so far yielded null results. Although it is quite possible that direct evidence for supersymmetry will be found at the upcoming “run 2” of the Tevatron collider, a decisive test of weak scale supersymmetry may only be possible at the LHC, which will not commence operations for another five or six years. In the meantime it is important to explore slightly less direct methods that might either give evidence for supersymmetry, or else constrain the (usually quite large) parameter space of supersymmetric models. Here we study two kinds of experiments which are expected to yield results with greatly improved sensitivity in the next couple of years: direct searches for ambient Dark Matter<sup>1</sup>, and the measurement of the anomalous dipole moment of the muon.

Current direct Dark Matter search experiments are sensitive to scenarios with  $\tilde{\chi}_1^0 p$  scattering cross section  $\sigma_{\chi p}$  of order  $10^{-6}$  pb or more. In fact, the DAMA experiment even claims evidence for WIMP Dark Matter, based on the annual modulation of their counting rate [6]; however, the CDMS experiment, which has comparable sensitivity, sees no signal [7]. Within the next two or three years we can expect results from the upgraded CDMS experiment (at the deep underground Soudan site) [8] as well as from the CRESST experiment in the Gran Sasso laboratory [9]. The sensitivity of these experiments should be about two orders of magnitude better than that of current experiments, so that scenarios with  $\sigma_{\chi p}$  exceeding  $10^{-8}$  pb might be testable.

SUSY models can also be tested using precision measurements at low energy experiments. Through their loop effects, sparticles contribute to low energy physics and these effects may become significant if the masses of sparticles are not too large. The supersymmetric contribution to the muon magnetic dipole moment (MDM)  $a_\mu = \frac{1}{2}(g-2)_\mu$  is one of the most robust probes. Unlike supersymmetric contributions to rare decays or CP violation in the  $B$ -system, which are currently being studied at the  $B$ -factories BaBar and BELLE, the supersymmetric contribution to  $a_\mu$  is not very sensitive to the way the supersymmetric flavor and CP problems are solved. At present, the muon MDM is measured to be  $a_\mu^{exp} = (116\,592\,05 \pm 45) \times 10^{-10}$  [10] and is consistent with the Standard Model. The ongoing analysis of data from the Brookhaven experiment E821 is expected to reduce the uncertainty further and the ultimate goal of the experiment is  $\Delta a_\mu \sim 4 \times 10^{-10}$  [11].<sup>2</sup> Supersymmetric contributions can easily exceed this sensitivity without violating any other constraints on the supersymmetric particle spectrum

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<sup>1</sup>Many physicists would accept direct detection of weakly interacting massive particles (WIMPs) as (component of) the dark halo of our galaxy as fairly direct evidence for weak-scale supersymmetry. However, given that direct detection experiments can basically only measure the mass and scattering cross section of the WIMP, such a detection, while undoubtedly of the greatest importance, may not convince everybody that supersymmetry has been found.

<sup>2</sup>In order to fully exploit such a small experimental error it is also necessary to reduce the hadronic uncertainty of the SM prediction, which currently stands at  $\sim 7 \times 10^{-10}$  [10, 12]. Here the analysis of data that is being collected at the low-energy  $e^+e^-$  colliders in Frascati, Novosibirsk and Beijing, as well as from semi-leptonic decays of the  $\tau$  lepton, will be crucial.

(from direct searches or other loop effects) [13].

These supersymmetric contributions to  $a_\mu$  stem from smuon–neutralino and sneutrino–chargino loops. Their size thus depends on the parameters that appear in the chargino and neutralino mass matrix, as well as on the masses of the sleptons. Of course, the parameters of the neutralino mass matrix also determine the mass and composition of the lightest neutralino, which we assume to form the Dark Matter in the Universe. Moreover, at least for the theoretically most appealing case of a bino–like LSP, the LSP relic density is essentially determined by the mass of the LSP as well as the masses of the  $SU(2)$  singlet sleptons [14]. The predictions for the relic density of bino–like LSPs and for the supersymmetric contribution to  $a_\mu$  are thus related.

There also exist relations between the SUSY contribution to  $a_\mu$  and the predicted LSP–nucleon scattering cross section. As already mentioned, both quantities depend on the parameters of the neutralino mass matrix. An even more direct connection comes from the common dependence on the ratio  $\tan\beta$  of the vevs of the two neutral Higgs fields of the MSSM, since both  $a_\mu$  and  $\sigma_{\chi p}$  increase with increasing  $\tan\beta$ . The operator in the effective Lagrangian that gives rise to the anomalous magnetic moment of the muon couples left– and right–handed muons, i.e. violates chirality. In the MSSM (as well as in the SM) all chirality violation in the (s)muon sector is proportional to the Yukawa coupling of the muon. This coupling in turn is proportional to  $1/\cos\beta$ , which scales like  $\tan\beta$  if  $\tan^2\beta \gg 1$ . The leading contributions to the spin–independent (coherent) contribution to  $\sigma_{\chi p}$  also involve violation of chirality [15], but this time in the (s)quark sector.<sup>3</sup> These contributions either come from the exchange of CP–even Higgs bosons, or from squark exchange. In both cases chirality violation ultimately comes from the quark Yukawa coupling, which, in case of down–type quarks, has the same  $\tan\beta$  dependence as the Yukawa coupling of the muon. Note that searches for Higgs bosons at LEP already imply  $\tan\beta \gtrsim 2$  in the MSSM<sup>4</sup>; this implies that the Yukawa couplings of up–type quarks, which scale  $\propto 1/\sin\beta$ , must be close to their SM–values, and essentially independent of  $\tan\beta$ . The upshot of this discussion is that both the SUSY contribution to  $a_\mu$  [13, 16] and  $\sigma_{\chi p}$  [17, 15, 18] can be quite large if  $\tan\beta \gg 1$ .

In this paper, we investigate these connections between the SUSY contribution  $\Delta a_\mu$  to the magnetic dipole moment of the muon and the spin–independent neutralino–proton cross section quantitatively. We study the minimal supergravity (mSUGRA) model as well as some other SUSY models where the assumption of strict scalar mass universality at the GUT scale is relaxed. In Sec. 2, we discuss  $\Delta a_\mu$  and  $\sigma_{\chi p}$  in the mSUGRA scenario. These observables are below the sensitivity of near–future experiments in the small  $\tan\beta$  region, partly because the experimental Higgs mass bounds then tightly restrict the allowed SUSY parameter space. However, they can become significant, and could be detectable, if  $\tan\beta$  is large. In Sec. 3 we relax the assumption of universal scalar masses. We find that the correlation between  $\Delta a_\mu$  and  $\sigma_{\chi p}$  quite sensitively depends on the boundary condition at the GUT scale. Sec. 4 is devoted to conclusions.

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<sup>3</sup>There are also squark exchange contributions where chirality violation is provided by the LSP mass, but these contributions are suppressed by an extra power of  $(m_q^2 - m_\chi^2)^{-1}$  compared to the leading squark exchange contribution, and can thus usually be ignored in the type of model we are considering [15].

<sup>4</sup>Higgs searches also permit  $\tan\beta \lesssim 0.6$ , but this is strongly disfavored on theoretical grounds, since the top Yukawa coupling would have a Landau pole quite close to the weak scale.

## 2 mSUGRA

In the minimal supergravity model, it is usually assumed that all squared scalar masses receive a common soft SUSY breaking contribution  $m_Q^2 = m_U^2 = m_D^2 = m_L^2 = m_E^2 = m_{H_u}^2 = m_{H_d}^2 \equiv m^2$  at the GUT scale  $M_X \simeq 2 \cdot 10^{16}$  GeV, while all gauginos receive a common mass  $M$  and all trilinear soft terms unify to  $A$ . The renormalization group (RG) evolution of soft breaking squared Higgs masses then leads to consistent breaking of the electroweak symmetry, provided the higgsino mass parameter  $\mu$  can be tuned independently [19]. In this paper, we chose the weak scale input parameters  $m_b(m_b) = 4.2$  GeV,  $m_t(m_t) = 165$  GeV, and  $\tan\beta$ . We minimize the tree level potential at renormalization scale  $Q = \sqrt{m_{\tilde{t}} m_t}$ , which essentially reproduces the correct value of  $\mu$  obtained by minimizing the full 1-loop effective potential [20].

With these assumptions, the mSUGRA model allows four continuous free parameters ( $m, M, A$  and  $\tan\beta$ ). Also, the sign of  $\mu$  remains undetermined. In Fig. 1, we plot the SUSY contribution to the muon MDM,  $\Delta a_\mu$ , vs. the spin-independent neutralino-proton cross section,  $\sigma_{\chi p}$ , for three different choices of  $\tan\beta = 4, 10$  and  $30$ . Here, we take  $A = 0$  and  $\mu > 0$  and allow  $m$  and  $M$  to vary in the intervals  $m < 500$  GeV and  $M < 600$  GeV. We include loop corrections to the masses of neutral Higgs bosons from third generation quarks and squarks, including leading two-loop corrections [21]. We use the expressions of ref.[16] for the calculation of  $\Delta a_\mu$ . The calculation of  $\sigma_{\chi p}$  is based on refs. [15, 22]. We use the value  $m_s \langle p | \bar{s}s | p \rangle = 130$  MeV for the strange quark's contribution to the nucleon mass; this matrix element is uncertain to about a factor of 2, leading to a similar uncertainty in the prediction of  $\sigma_{\chi p}$ . Finally, the calculation of the scaled LSP relic density  $\Omega_\chi h^2$  uses results of refs.[14, 23];  $s$ -channel poles are treated as described in ref.[24]. The co-annihilation of  $\tilde{\chi}_1^0$  with sleptons is not included. This effect can increase the cosmologically allowed region of parameter space towards higher LSP masses, if  $m_{\tilde{\chi}_1^0} \simeq m_{\tilde{l}}$ ; however, in this (limited) region of parameter space both  $\Delta a_\mu$  and  $\sigma_{\chi p}$  are well below present and near-future sensitivity.

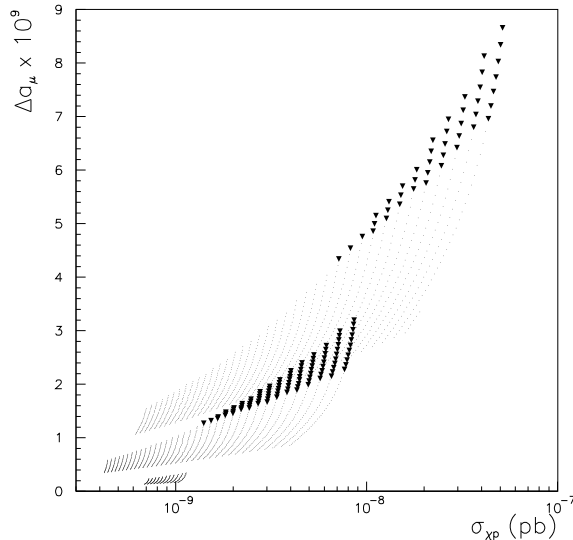


Figure 1:  $\Delta a_\mu$  vs.  $\sigma_{\chi p}$  in mSUGRA for  $\tan\beta = 4, 10, 30$  (from bottom to top region). We take  $A = 0$  and  $\mu > 0$ , and scan  $m \in [0, 500]$  GeV and  $M \in [0, 600]$  GeV, subject to experimental constraints. The heavily marked points satisfy the requirement  $\Omega_\chi h^2 < 0.3$ .

Accelerator bounds significantly limit the SUSY parameter space. For the parameter range scanned in this plot we find that the lightest Higgs boson  $h$  couples essentially like the single Higgs boson of the SM; we thus demand that its mass  $m_h > 111$  GeV. This follows from recent LEP results [25], allowing for a 2 GeV theoretical uncertainty in the calculation of  $m_h$  [21]. We further require that the chargino mass  $m_{\tilde{\chi}_1^\pm} > 100$  GeV [26]. We exclude regions where the LSP is charged, i.e. where  $m_{\tilde{\tau}_1} < m_{\tilde{\chi}_1^0}$  or  $m_{\tilde{t}_1} < m_{\tilde{\chi}_1^0}$ . The heavily marked points satisfy the further requirement  $\Omega_\chi h^2 < 0.3$ . This bound on the matter density in the Universe follows from the analysis of recent cosmological data [27].

The lowest dotted region in Fig. 1 corresponds to  $\tan\beta = 4$ . In this case both  $\Delta a_\mu$  and  $\sigma_{\chi p}$  are too small to be detected in the near future. Furthermore, the lower bound on the Higgs mass forces one into a region of parameter space where the dark matter density is quite large and therefore cosmologically disfavored. Through radiative corrections, which increase with increasing stop masses,  $m_h$  depends quite strongly on  $M$ , but only weakly on  $m$ . The reason is that contributions from Yukawa interactions to the RG equations reduce scalar masses at the weak scale as compared to their GUT scale values. As a result, in the expressions for the weak scale squared stop masses  $m_{\tilde{t}_L}^2$  and  $m_{\tilde{t}_R}^2$ ,  $m^2$  appears with coefficients significantly less than 1, while  $M^2$  appears with coefficients well above 1 [19]. For  $\tan\beta = 4$ , the Higgs mass bound  $m_h > 111$  GeV requires  $M > 500$  GeV, which corresponds to  $m_{\tilde{\chi}_1^0} > 217$  GeV. Such a large SUSY mass scale gives too large a DM mass density [14].

As  $\tan\beta$  increases, both  $a_\mu$  and  $\sigma_{\chi p}$  increase. As mentioned above there are essentially two types of diagrams involving superparticles which contribute to  $a_\mu$ , i.e., neutralino–smuon and chargino–sneutrino loop diagrams. Since  $a_\mu$  requires chirality violation, for  $\tan\beta \gg 1$  the dominant contributions are proportional to the product of an electroweak gauge coupling and the Yukawa coupling of the muon, where the latter factor either comes directly from the higgsino component of the chargino or neutralino in the loop, or from  $\tilde{\mu}_L - \tilde{\mu}_R$  mixing. For  $\tan\beta \gg 1$  one thus has  $\Delta a_\mu \propto \tan\beta$ . If mass splittings between different sparticles are not too large, so that their mass scale can be described by the single parameter  $m_{\text{SUSY}}$ , the total result can be estimated as [16]

$$|\Delta a_\mu| = \frac{1}{32\pi^2} \left( \frac{5}{6}g_2^2 + \frac{1}{6}g_1^2 \right) \frac{m_\mu^2}{m_{\text{SUSY}}^2} \tan\beta \quad (1)$$

where  $g_1$  and  $g_2$  are the  $U(1)_Y$  and  $SU(2)$  gauge couplings, respectively. In our convention, where the gaugino masses and  $\tan\beta$  are positive, the sign of  $\Delta a_\mu$  is equal to the sign of  $\mu$ .

On the other hand, three classes of diagrams contribute to LSP–quark scattering: the exchange of a  $Z$  or CP–even Higgs boson in the  $t$  channel and squark exchange in the  $s$  or  $u$  channel. LSP interactions with matter can naturally be separated into spin–dependent and spin–independent parts.  $Z$  and squark exchange contribute to the former, and squark and Higgs exchange to the latter. In most situations the dominant contribution to the spin independent amplitude is the exchange of the two neutral CP–even Higgs bosons, although in some cases the contribution of squark exchange is substantial [15]. For given inputs  $m, M, A$  and  $\text{sign}(\mu)$ , the mass of the heavy CP–even Higgs boson decreases as  $\tan\beta$  increases. This effect originates from the contribution of the bottom Yukawa coupling to the RGE of  $m_{H_d}^2$  [28]; recall that this Yukawa coupling also scales like  $\tan\beta$  for  $\tan\beta \gg 1$ . Moreover, its coupling to  $d$ –type quarks is essentially proportional to  $\tan\beta$  for  $\tan^2\beta \gg 1$  and  $m_A^2 > m_{h,\text{max}}^2$ , where  $m_A$  is the mass of the CP–odd Higgs boson and  $m_{h,\text{max}}^2$  is the  $m_A \rightarrow \infty$  limit of the mass of the light CP–even Higgs boson. Under these conditions, which are almost always satisfied in the models we are studying,  $h$  couples essentially like the single Higgs boson of the SM, independent of the value of  $\tan\beta$ ;  $m_h$  also becomes almost independent of  $\tan\beta$  for  $\tan\beta \gtrsim 10$ . The contribution

to  $\sigma_{\chi p}$  from  $H$  exchange, which grows quickly with increasing  $\tan\beta$ , thus starts to dominate over that from  $h$  exchange once  $\tan\beta \gtrsim (m_H/m_h)^2$ . Moreover, the leading down-type squark exchange contribution to  $\sigma_{\chi p}$ , which requires chirality breaking in the (s)quark sector, also increases  $\propto \tan\beta$  for large  $\tan\beta$ .

We can see these enhancements in Fig. 1. For  $\tan\beta = 10$ ,  $\Delta a_\mu$  is, in the cosmologically favored region, well above  $1 \times 10^{-9}$  and can reach upto  $\sim 3 \times 10^{-9}$ , which could be detected in the near future. Also,  $\sigma_{\chi p}$  increases quite a lot, but still remains below  $10^{-8}$  pb. These enhancements, compared to the  $\tan\beta = 4$  case, partly come from the lower SUSY mass scale that is permitted by the Higgs search limit. For  $\tan\beta = 10$ , the Higgs mass bound  $m_h > 111\text{GeV}$  only implies  $M > 260\text{ GeV}$ , which corresponds to  $m_{\tilde{\chi}_1^0} > 110\text{ GeV}$ ; this allows scenarios with cosmologically favored DM mass density in the low  $m$  and  $M$  region. For  $\tan\beta = 30$ , in the cosmologically favored region  $\Delta a_\mu$  takes values of  $4 \sim 9 \times 10^{-9}$ , while  $\sigma_{\chi p}$  is mostly above  $1 \times 10^{-8}$  pb and can reach upto  $5 \times 10^{-8}$  pb. The smallest allowed SUSY mass scale for  $\tan\beta = 30$  is only slightly lower than that for  $\tan\beta = 10$ , since, as stated above,  $m_h$  becomes quite insensitive to  $\tan\beta$  for  $\tan\beta \gtrsim 10$ .

Note that the highest  $\Delta a_\mu$  values shown in Fig. 1 already reach the *current* level of sensitivity, whereas the maximal value of  $\sigma_{\chi p}$  in this figure is still more than an order of magnitude below current sensitivity. We conclude that, in view of near future detectability,  $\Delta a_\mu$  is more sensitive than  $\sigma_{\chi p}$  in the mSUGRA model; this agrees with results of ref.[29] (for  $m \leq 500\text{ GeV}$ ). One of the important factors for determining the size of  $\sigma_{\chi p}$  is the higgsino component of the LSP. MSUGRA predicts a bino-like LSP  $\tilde{\chi}_1^0$  for moderate values of  $m$  and  $M$  (below  $\sim 500\text{ GeV}$ ) [30, 14]. This is a rather model independent result [31]. Large positive corrections to squark masses from gaugino loops, together with the large top Yukawa coupling, drive the squared soft breaking Higgs mass  $m_{H_u}^2$  negative at the weak scale. On the other hand, correct symmetry breaking requires  $m_{H_u}^2 + \mu^2 > -M_Z^2/2$ . One has to make  $|\mu|$  large in order to obtain the correct electroweak symmetry breaking scale, if scalar masses and gaugino masses are of the same order. Since the Higgs–LSP–LSP couplings require higgsino–gaugino mixing, they scale like  $1/\mu$  for  $\mu^2 \gg M_Z^2$ . The Higgs exchange contribution to  $\sigma_{\chi p}$  then scales like  $1/\mu^2$ . Similarly, the prediction for  $\Delta a_\mu$  depends on  $\mu$  through gaugino–higgsino mixing as well as  $\tilde{\mu}_L - \tilde{\mu}_R$  mixing. The results of Fig. 1 thus depend quite sensitively on the value of  $\mu$  at the weak scale, which in turn depends on the GUT scale boundary conditions. The correlation between  $\Delta a_\mu$  and  $\sigma_{\chi p}$  could also be different in generalizations of mSUGRA. That is the subject of the next section.

Before going to the next section, we wish to discuss the dependence of our results on the parameter  $A$  as well as the sign of  $\mu$ . The  $A$ –dependence is mostly indirect. First of all,  $A$  has some impact on  $m_h$ , and hence on the allowed region of the  $(m, M)$  plane. This can be seen in Fig. 2 a), where the Higgs mass dependence on  $M$  is plotted for three different choices of  $A = +2M, 0, -2M$ . For a given  $M$ , a larger  $A$  gives larger  $m_h$ . Therefore, if we chose a large positive  $A$  value, the minimal allowed value of  $M$  will be lowered, so that the maximal allowed value of  $\Delta a_\mu$  will be raised, even though this observable is almost independent of  $A$  if the slepton and –ino mass parameters are held fixed.  $A$  also has some impact on the  $\mu$  parameter. As we can see in Fig. 2 b), larger  $A$  values give higher  $\mu$  values and therefore more bino-like LSP for a given  $M$ . This reduction of the higgsino component of the LSP would reduce the Higgs exchange contribution to  $\sigma_{\chi p}$ . Some numerical examples of the effects of varying  $A$  are given in the next section.

Changing the sign of  $\mu$  from positive to negative has both indirect and direct effects. Since now  $A$  and  $\mu$  appear with opposite sign in the  $\tilde{t}_L - \tilde{t}_R$  mixing terms, the predicted value of  $m_h$  is reduced slightly. One thus has to increase the minimal allowed value of  $M$  in order

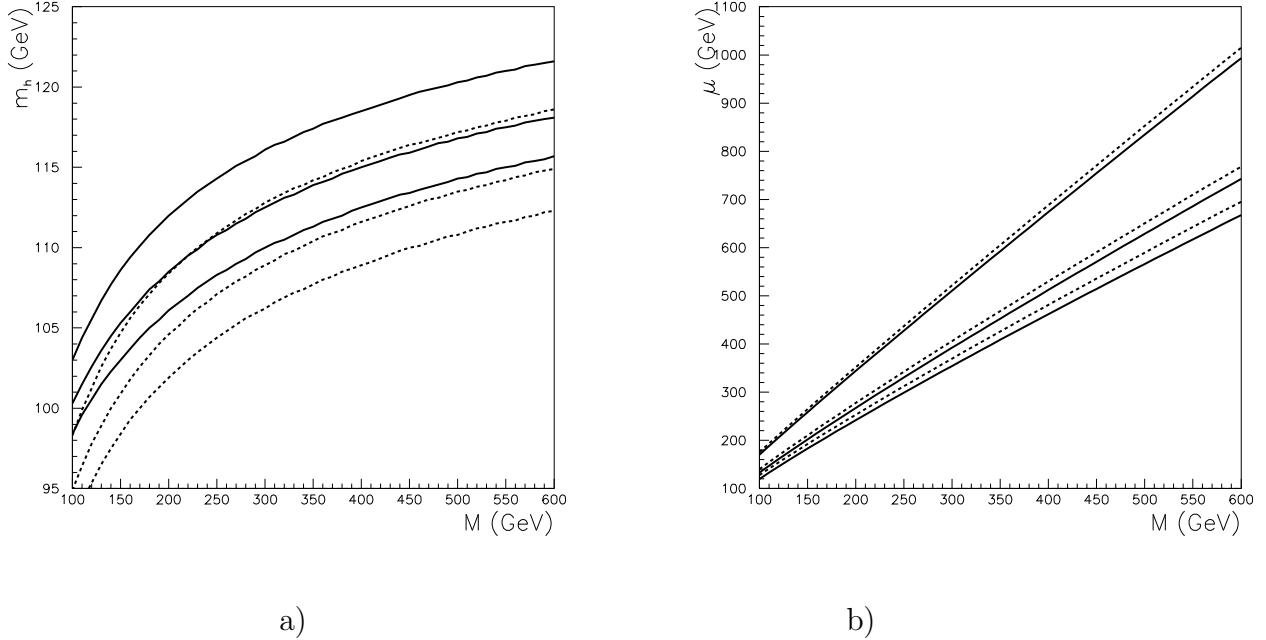


Figure 2: The dependence of a)  $m_h$  and b)  $\mu$  on  $M$  for three different choices of  $A = +2M, 0, -2M$  (from top to bottom). Here we fix  $m=100$  GeV for definiteness. The dotted and solid lines are for  $\tan\beta = 5, 10$  respectively.

to satisfy the bound  $m_h > 111$  GeV. In fact, we didn't find any allowed solutions within the scanning range for  $\tan\beta = 4$ ,  $A_0 = 0$  and  $\mu < 0$ , while the minimal allowed value of  $M$  for  $\tan\beta = 10$  is increased by  $\sim 40$  GeV, leading to a reduction of the maximal possible  $|\Delta a_\mu|$  to about  $2.4 \times 10^{-9}$ . Note that for fixed values of the other parameters, the magnitude of  $\Delta a_\mu$  is almost independent of the sign of  $\mu$ , but the sign of  $\Delta a_\mu$  changes when the sign of  $\mu$  is flipped. In contrast, the sizes of the  $\tilde{\chi}_1^0 \tilde{\chi}_1^0(h, H)$  couplings do depend quite significantly on the sign of  $\mu$ , unless  $\tan\beta \gg 1$  [14]. In particular, for  $\mu < 0$  strong cancellations occur [15] both within different contributions to the same coupling, and between the  $h$  and  $H$  exchange contributions to  $\sigma_{\chi p}$ . As a result, for  $\tan\beta = 10$ , cross sections as low as  $4 \times 10^{-11}$  pb are possible in the cosmologically favored region, and the maximal allowed value of  $\sigma_{\chi p}$  is reduced by more than one order of magnitude, to  $5 \times 10^{-10}$  pb. The importance of the sign of  $\mu$  diminishes with increasing  $\tan\beta$ ; hence results for  $\tan\beta = 30$  and  $\mu < 0$  are quite similar to the results for  $\tan\beta = 30$  shown in Fig. 1. Finally, we mention that in models with universal scalar masses, solutions with  $\mu < 0$  tend to have too large a branching ratio for radiative  $b \rightarrow s\gamma$  decays [32].<sup>5</sup> However, this prediction is sensitive to details of the flavor structure of the soft breaking terms, unlike the quantities we have depicted in Fig. 1. We therefore do not attempt to analyze the constraint from  $b \rightarrow s\gamma$  decays quantitatively.

<sup>5</sup>Recently the leading higher order corrections to this branching ratio have been computed [33]. While a detailed parameter scan has not yet been performed, the published results indicate that scenarios with degenerate scalar masses and  $\mu < 0$  remain problematic at large  $\tan\beta$ . On the other hand, for  $\mu > 0$  agreement with the data can be obtained even for  $\tan\beta \simeq 45$ .

### 3 More general models

In this section we relax our assumptions, allowing for non-universal soft scalar masses at the GUT scale, while keeping the unification of the gaugino masses. As specific models, we consider a string-inspired supergravity model and an  $SO(10)$  Grand Unified model.

Soft SUSY breaking terms have been studied within the framework of string-inspired supergravity models [34]. In a wide class of models one obtains the following relations among soft SUSY breaking terms:

$$A_{ijk} = -M, \quad m_i^2 + m_j^2 + m_k^2 = M^2, \quad (2)$$

for non-vanishing Yukawa couplings  $Y_{ijk}$  of chiral superfields  $\Phi^i$ ,  $\Phi^j$  and  $\Phi^k$ . These relations hold if the theory is target-space duality-invariant and the Yukawa couplings in the supergravity basis are field-independent. In addition we assume that only the  $F$ -terms of the dilaton and the moduli superfields contribute to SUSY breaking, and that the vacuum energy vanishes. The generic form of supergravity theories leading to these relations has been obtained in Ref. [35].<sup>6</sup> These relations allow non-universality of soft scalar masses, but their overall magnitudes are bounded by the gaugino mass through the sum. This point is significant.

Here we apply the relations (2) to the up and down sectors of (s)quarks and the (s)lepton sector separately. To be explicit, we have the relations between the soft breaking contributions to the squared scalar masses,

$$\begin{aligned} m_Q^2 + m_U^2 + m_{H_u}^2 &= M^2 \\ m_Q^2 + m_D^2 + m_{H_d}^2 &= M^2 \\ m_L^2 + m_E^2 + m_{H_d}^2 &= M^2, \end{aligned} \quad (3)$$

and a universal  $A$ -term,  $A = -M$ .

In fact, these relations include quite a large parameter space, where sfermion masses are not equal to each other. However, as the first trial we take universal sfermion masses; this remains the simplest solution of the supersymmetric flavor problem. We shall later comment on a case with non-universal sfermion masses.

The  $SO(10)$  theory incorporates a complete generation of MSSM matter superfields into the 16-dimensional spinor representation,  $\Psi_{16}$ . In addition to these matter superfield, the minimal  $SO(10)$  model includes a 10-dimensional Higgs superfield  $\Phi_{10}$  which contains the two Higgs superfields of the MSSM (as well as their  $SU(3)$  triplet,  $SU(2)$  singlet partners). When  $SO(10)$  breaks to the MSSM gauge group  $SU(3)_C \times SU(2)_L \times U(1)_Y$ , additional  $D$ -term contributions (parameterized by  $M_D^2$  which can be either positive or negative) to the soft SUSY breaking masses arise [36]:

$$\begin{aligned} m_Q^2 &= m_E^2 = m_U^2 = m_{16}^2 + M_D^2 \\ m_D^2 &= m_L^2 = m_{16}^2 - 3M_D^2, \quad m_{H_{u,d}}^2 = m_{10}^2 \mp 2M_D^2, \end{aligned} \quad (4)$$

where  $m_{16}$  and  $m_{10}$  are scalar soft breaking masses for fields in the **16** and **10** dimensional representations of  $SO(10)$ , respectively.

The modifications (3) and (4) of the mSUGRA boundary conditions change our predictions in two different ways: through modifications of the slepton spectrum (which change  $\Delta a_\mu$  and  $\Omega_\chi h^2$ ), and through the changed value of  $|\mu|$  at the weak scale, which mostly affects  $\sigma_{\chi p}$ . While

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<sup>6</sup>These relations are RG-invariant at one-loop for a single gauge group.



the changes of the slepton spectrum can be read off directly from eqs.(3) and (4), the changes of  $|\mu|$  are more subtle. The main effects can be understood as follows. In the mSUGRA scenario, the contributions of  $m$  and  $M$  to the weak scale values of the soft breaking Higgs boson masses can be parameterized as

$$\begin{aligned} m_{H_d}^2 &\simeq m^2 + 0.5M^2; \\ m_{H_u}^2 &\simeq \epsilon_H m^2 - 3.3M^2, \end{aligned} \quad (5)$$

where we have assumed  $\sin \beta \simeq 1$  but ignored contributions from the bottom Yukawa coupling. The coefficient  $\epsilon_H$  is small, because the GUT scale value of  $m_{H_u}^2$  is canceled by the scalar masses appearing in the RG running.<sup>7</sup> The effect of non-universality on  $m_{H_u}^2$  can be parameterized by introducing  $m_Q^2 + m_U^2 + m_{H_u}^2 = 3m_s^2$  and  $\delta m_H^2 = m_{H_u}^2 - m_s^2$ :

$$m_{H_u}^2 = \delta m_H^2 + \epsilon_H m_s^2 - 3.3M^2. \quad (6)$$

This equation follows because the radiative correction to  $m_{H_u}^2$  is proportional to  $m_Q^2 + m_U^2 + m_{H_u}^2$ , hence the effect of RGE running is the same as in mSUGRA with the replacement  $m^2 \rightarrow m_s^2$ . If  $\sin \beta \simeq 1$ , correct gauge symmetry breaking requires  $\mu^2 \simeq -m_{H_u}^2 - M_Z^2/2$ , where all quantities are taken at the weak scale. Hence  $\delta m_H^2$  directly affects the value of  $|\mu|$  at the weak scale. For the string scenario (3) the effect is limited because the total scalar mass is bounded from above by the gaugino mass, but for the  $SO(10)$  scenario (4) the change of  $|\mu|$  from its mSUGRA prediction can be more drastic.

Fig. 3 shows the correlation between  $\Delta a_\mu$  and  $\sigma_{\chi p}$  for  $\tan \beta = 10$  and  $\mu > 0$  in three different models: mSUGRA, the string scenario described by eq.(3) with universal sfermion masses ( $\equiv m$ ), and the  $SO(10)$  GUT model. Note that the assumption of universal sfermion masses implies  $m_{H_d}^2 = m_{H_u}^2$  at the GUT scale in the stringy model; the value of the Higgs soft breaking masses is determined by eq.(3) for given  $m$  and  $M$ , and  $A = -M$ . For the  $SO(10)$  GUT model, we assume  $m_{16} = m_{10} \equiv m$  and fix  $A = 0$  and  $M_D^2 = -50000 \text{ GeV}^2$ . As before, we scan the region  $m < 500 \text{ GeV}$  and  $M < 600 \text{ GeV}$ , requiring the physical Higgs mass and chargino mass to lie above their experimental lower bounds, and excluding regions with charged LSP. The heavily marked points again indicate the cosmologically favored region where  $\Omega_\chi h^2 < 0.3$ .

In our superstring scenario the higgsino component of the LSP can be larger or smaller than in mSUGRA, depending on the ratio  $m/M$ . For fixed  $M$ , a small  $m$  gives a larger higgsino component of the LSP because then  $m_{H_{u,d}}^2$  should be larger than  $m^2$  in order to satisfy eq.(3), i.e.  $\delta m_H^2 > 0$  in eq.(6). This in turn reduces the supersymmetric contribution  $\mu^2$  to the Higgs mass term which is required to obtain the correct  $Z$  boson mass. On the other hand, for  $m^2 > M^2/3$ , eq.(3) gives  $\delta m_H^2 < 0$ , leading to even larger  $|\mu|$  than in mSUGRA. Altogether this model allows for a broader range of values of  $\sigma_{\chi p}$ , with maximum around  $1.7 \times 10^{-8} \text{ pb}$ , about a factor of 2 above the maximal mSUGRA prediction. On the other hand, the maximum value of  $\Delta a_\mu$  is reduced to  $2.3 \times 10^{-9}$  from  $3.3 \times 10^{-9}$  in mSUGRA. This reduction mainly comes from the fact that the minimal allowed value of  $M$  is raised by about 40 GeV due to the choice  $A = -M$  (rather than  $A = 0$  in the mSUGRA case); we saw in Fig. 2 that negative values of  $A$  tend to reduce  $m_h$  if all other input parameters are kept fixed. The variation of  $\mu$  is less important for  $\Delta a_\mu$ . The reason is that increasing  $|\mu|$  increases the higgsino mass and

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<sup>7</sup>The effective  $\epsilon_H$  is slightly negative for low SUSY breaking scale, but turns positive if this scale is large. Recall that the relevant scale for the analysis of gauge symmetry breaking increases with increasing sparticle masses.

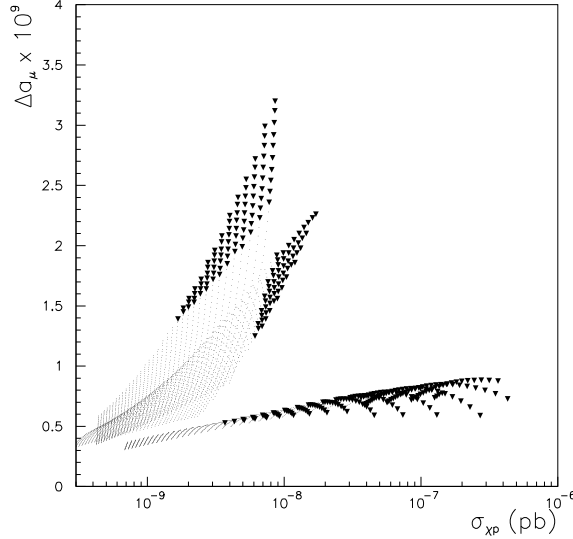


Figure 3:  $\Delta a_\mu$  vs.  $\sigma_{xp}$  in three different models : mSUGRA, a superstring scenario and the minimal  $SO(10)$  GUT model. We chose  $\tan\beta = 10$  and  $\mu > 0$ . The scanning procedure and notation is as in Fig. 1.

reduces higgsino–gaugino mixing, which reduces  $|\Delta a_\mu|$ , but at the same time increases  $\tilde{\mu}_L - \tilde{\mu}_R$  mixing, which tends to increase  $|\Delta a_\mu|$ .

We also performed a scan (not shown) for a second ansatz for scalar soft breaking masses which is compatible with the general expressions (3). Here we kept the soft breaking contributions to the squared squark and Higgs boson masses equal to each other. This implies  $m_Q^2 = m_U^2 = m_D^2 = m_{H_u}^2 = m_{H_d}^2 = M^2/3$  and  $m_E^2 + m_L^2 = 2M^2/3$ . We then allowed  $m_E$  and  $m_L$  to differ from each other, keeping universality among different generations. This scan gives results for  $\sigma_{xp}$  which are quite similar to those of the mSUGRA scenario; however, the predictions for  $a_\mu$  are generally lower than in mSUGRA. This is true in particular for parameter values that satisfy  $\Omega_\chi h^2 < 0.3$ . For the given relation between average slepton mass and gaugino mass, an acceptable value of the relic density can only be achieved<sup>8</sup> if  $m_E < m_L$ ; note that due to its larger hypercharge,  $\tilde{e}_R$  exchange contributes 8 times more strongly to the annihilation of bino–like LSPs than  $\tilde{e}_L$  plus  $\tilde{\nu}$  exchange does. This mass splitting between  $\tilde{e}_R$  and  $\tilde{\nu}$  reduces  $\Delta a_\mu$ , since the (usually dominant) sneutrino–chargino loop contributions are suppressed compared to the scenario with universal slepton masses. The main difference to our first stringy scenario, where we allowed different soft breaking contributions for sfermions and Higgs bosons while keeping sfermion masses equal to each other, are the lower maximally allowed values of both  $\sigma_{xp}$  and  $\Delta a_\mu$ . This also leads to smaller values of  $\Delta a_\mu$  and  $\sigma_{xp}$  in the cosmologically acceptable region: the minimal  $\sigma_{xp}$  which is compatible with  $\Omega_\chi h^2 < 0.3$  is reduced to  $\sim 2.5 \times 10^{-9}$  pb from  $\sim 6 \times 10^{-9}$  pb, while the smallest cosmologically allowed  $\Delta a_\mu$  is reduced from  $\sim 1.25 \times 10^{-9}$  to  $\sim 0.9 \times 10^{-9}$ .

While the predictions of these versions of the superstring inspired model differ from those of mSUGRA “only” by about a factor of 2, the  $SO(10)$  GUT model with  $M_D^2 = -50000 \text{ GeV}^2$

<sup>8</sup>We note in passing that the dilaton–dominated scenario [37], where all scalar soft breaking masses are equal to  $M/\sqrt{3}$ , cannot simultaneously accommodate  $m_h > 111 \text{ GeV}$  and  $\Omega_\chi h^2 < 0.3$ , unless  $\tan\beta$  is very large.

leads to a dramatically changed correlation between  $\Delta a_\mu$  and  $\sigma_{\chi p}$ . Since  $m_{H_u}^2$  at the GUT scale is larger than in mSUGRA, the higgsino component of the LSP increases. This effect is especially significant for small  $M$ , where the gaugino mass contributions to the RGE of the scalar masses are not so large, so that a modification of the GUT scale boundary condition more strongly affects weak scale values of scalar masses; see eq.(6). In this region  $\sigma_{\chi p}$  increases quite a lot, reaching values up to  $\sim 4 \times 10^{-7}$  pb, which could be detected in the near future. In contrast, the allowed values of  $\Delta a_\mu$  are less than  $1 \times 10^{-9}$ , well below near future sensitivity. This reduction is caused by the increase of the sneutrino mass and the decrease of the  $\tilde{\mu}_R$  mass due to the  $D$ -term contributions in eq.(4). Because of this large slepton mass splitting, eq.(1) is not applicable in this scenario. The  $D$ -terms suppress the contribution from chargino-sneutrino loops, while enhancing the contribution from neutralino-smuon loops. These two contributions can now have comparable magnitude, but opposite sign, leading to significant cancellations in the small  $m$  region. These cancellations explain why we sometimes find an anti-correlation between  $\sigma_{\chi p}$  and  $\Delta a_\mu$  in this model, i.e. decreasing  $m^2$  can lead to decreasing  $\Delta a_\mu$  (and increasing  $\sigma_{\chi p}$ , as usual). Of course, for large  $m$  all contributions to  $\Delta a_\mu$  are small.

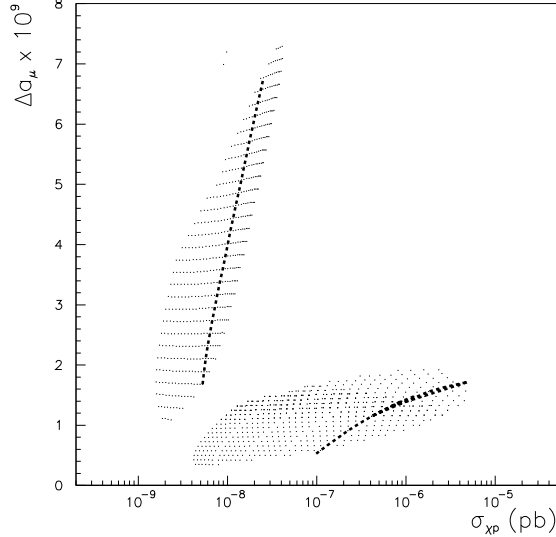


Figure 4:  $\Delta a_\mu$  vs.  $\sigma_{\chi p}$  in mSUGRA and the  $SO(10)$  model with  $M_D^2 = -50000 \text{ GeV}^2$ . Here, we fix  $m$  and  $M$  such that  $m_{\tilde{\chi}_1^0} \simeq 120 \text{ GeV}$  and  $m_{\tilde{e}_R} \simeq 182 \text{ GeV}$ , and vary  $A$  and  $\tan\beta$ .

In Fig. 3 the models with and without  $SO(10)$   $D$ -terms are well separated. Therefore one might conclude that one can test for the existence of  $SO(10)$   $D$ -terms by measuring  $\Delta a_\mu$  and/or  $\sigma_{\chi p}$ . However, in Fig. 3 we fixed the values of  $\tan\beta$ ,  $A$  and  $M_D^2$ . In order to see the effects of varying  $\tan\beta$  and  $A$ , we fix  $m = 140 \text{ GeV}$  and  $M = 280 \text{ GeV}$  in mSUGRA, and  $m = 260 \text{ GeV}$  and  $M = 300 \text{ GeV}$  in the  $SO(10)$  model, so that  $m_{\tilde{\chi}_1^0} \simeq 120 \text{ GeV}$  and  $m_{\tilde{e}_R} \simeq 182 \text{ GeV}$  in both cases. We then scan the region of  $-2M < A < 2M$  and the entire allowed range of  $\tan\beta$ . Both the lower and upper bounds on  $\tan\beta$  come from Higgs searches at LEP; recall that in models with radiative symmetry breaking, *all* neutral Higgs bosons become quite light as  $\tan\beta$  approaches its upper bound [28]. Fig. 4. shows the result of this scan. Here the short-dashed line corresponds to  $A = 0$ , with positive (negative) values of  $A$  falling to the left

(right) side of this line. As expected,  $\Delta a_\mu$  increases when  $\tan\beta$  increases, and depends only weakly on  $A$ . On the other hand,  $\sigma_{\chi p}$  increases with increasing  $\tan\beta$ , but decrease when  $A$  is increased. As noted earlier, this reduction of  $\sigma_{\chi p}$  comes from the fact that the Higgs masses increase, and the LSP becomes more bino-like, when  $A$  increases.

In this figure, mSUGRA scenarios are characterized by relatively large  $\Delta a_\mu$  and small  $\sigma_{\chi p}$ , but the opposite is true for the  $SO(10)$  GUT model with  $M_D^2 = -50000 \text{ GeV}^2$  in most of the allowed region. The two classes of models thus remain quite well separated. Of course, it is by no means certain that the extra  $D$ -term contribution in the  $SO(10)$  model should be large and negative. In fact, if one requires that all three third generation Yukawa couplings unify at the GUT scale, which is not unreasonable in minimal  $SO(10)$  since all third generation superfields reside in a single representation of  $SO(10)$ , consistent radiative symmetry breaking is only possible if  $M_D^2 > 0$  [38]. In Figs. 5a, b we therefore show the dependence of  $\Delta a_\mu$  and  $\sigma_{\chi p}$  on  $M_D^2$ , for  $m = 260 \text{ GeV}$ ,  $M = 300 \text{ GeV}$ ,  $A = 0$  and  $\tan\beta = 10$ . The upper and lower bounds on  $M_D^2$  come from the requirement that  $SU(2)$  singlet and doublet sleptons, respectively, should be heavier than the lightest neutralino. We see that  $\sigma_{\chi p}$  increases very quickly for negative  $M_D^2$ ; near the lower bound on  $M_D^2$  one even approaches the sensitivity of *current* Dark Matter search experiments. Conversely,  $\Delta a_\mu$  increases if  $M_D^2$  is raised above 0, but the maximal increase amounts to less than a factor of 2 compared to the mSUGRA situation, which corresponds to  $M_D^2 = 0$ . Note, however, that Yukawa unification also requires  $\tan\beta \gtrsim 35$  [38]; the combined enhancement of  $\Delta a_\mu$  from large  $\tan\beta$  and from  $M_D^2 > 0$  implies that measurements of the anomalous dipole moment of the muon might be one of the best ways to test  $SO(10)$  models with Yukawa unification.

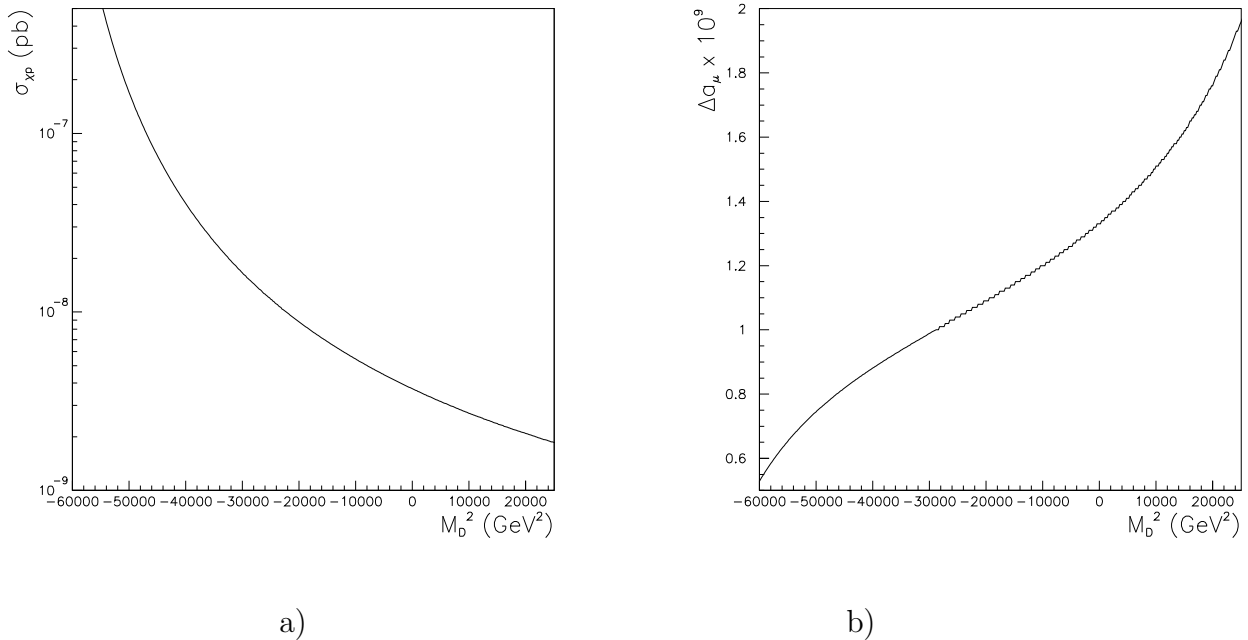


Figure 5: The dependence of a)  $\sigma_{\chi p}$  and b)  $\Delta a_\mu$  on  $M_D^2$  in the  $SO(10)$  model, for  $m = 260 \text{ GeV}$ ,  $M = 300 \text{ GeV}$ ,  $A = 0$  and  $\tan\beta = 10$ .

## 4 Conclusions

In this paper, we investigated the SUSY contribution to the muon MDM  $a_\mu$ , and to the spin-independent neutralino-proton cross section  $\sigma_{\chi p}$ , in several SUSY models with either universal or non-universal soft scalar masses at the GUT scale. Both these observables can become significant if  $\tan\beta$  is large, but will be difficult to measure in the small  $\tan\beta$  region. Both quantities are sensitive to chirality violation in the matter (s)fermion sector, which in the MSSM is enhanced for large  $\tan\beta$ . Conversely, if  $\tan\beta$  is small the experimental Higgs mass bounds tightly restrict the allowed SUSY parameter space, forcing the SUSY breaking scale to be quite high. While these general statements are fairly model-independent as long as one sticks to the field content of the MSSM, quantitative predictions do depend significantly on details of the spectrum of superparticles, in particular on the implementation of radiative gauge symmetry breaking.

We considered three different models: mSUGRA, a superstring scenario where SUSY is broken by the  $F$ -terms of the dilaton and the moduli superfields, and an  $SO(10)$  GUT model. The superstring model in principle has a very large parameter space, since it allows for non-universal masses for matter sfermions and Higgs bosons, subject only to the sum rule (2). Here we explored two orthogonal directions in this parameter space, where universality is violated either only in the Higgs sector or only in the slepton sector. We found that allowing the Higgs masses to differ from the sfermion masses at the GUT scale can vary the predicted value of  $\sigma_{\chi p}$  by a factor of a few in either direction, compared to the mSUGRA prediction for the same parameters. Given the uncertainty in the prediction of  $\sigma_{\chi p}$  due to unknown hadronic matrix elements, and the uncertainty in the predicted LSP-nucleus scattering rate due to the uncertainty of the ambient LSP flux, such a variation is barely significant. The prediction of  $\Delta a_\mu$  is almost insensitive to this variation of the Higgs soft breaking masses. Conversely, if we maintain the universality of the soft breaking contributions to squark and Higgs masses but allow the masses of  $SU(2)$  doublet and singlet sleptons to differ, we can change the prediction for  $\Delta a_\mu$  by a factor of  $\sim 2$  around the mSUGRA value, while keeping  $\sigma_{\chi p}$  constant; the predicted LSP relic density, and hence the cosmologically preferred region of parameter space, also depends on this ratio of slepton masses. Generally speaking, in mSUGRA as well as its stringy variant the measurement of the anomalous dipole moment of the muon seems to hold more promise in the near future than searches for LSP Dark Matter; this is largely due to the most recent severe constraints on the Higgs sector from LEP experiments.

We should caution the reader that this conclusion crucially depends on our choice to only study scenarios where both squarks and gluinos can be produced at the LHC. Given current slepton and chargino search limits, this implies that sfermion and gaugino masses are very roughly of the same order of magnitude. If one allows scalar masses  $m \gtrsim 1$  TeV and  $m^2 \gg M^2$ , one can find solutions in mSUGRA where the LSP has a large, or even dominant, higgsino component [39]. Since in this region of parameter space all sfermions are very heavy, the supersymmetric contribution to  $a_\mu$  is totally negligible, but  $\sigma_{\chi p}$  can be very large [40].

Much more dramatic deviations from mSUGRA predictions are possible if one introduces  $SO(10)$   $D$ -term contributions to scalar masses. In this model the maximal allowed value of  $\sigma_{\chi p}$  for given  $\tan\beta$  can exceed the mSUGRA prediction by more than a factor of 30, if the  $D$ -term contribution to the mass of the Higgs boson that couples to the top quark is positive. For the same sign of the  $D$ -terms the supersymmetric contribution to  $a_\mu$  is reduced by a factor of a few, since the change of the slepton masses increases the importance of neutralino loop diagrams compared to chargino loops, leading to a strong cancellation in the total result. In this kind of model direct Dark Matter searches therefore appear to be more promising in the next few years than measurements of  $a_\mu$ .

A striking result of our analysis is that even if we restrict squark and gluino masses to lie within discovery range of the LHC, and in addition fix the value of  $\tan\beta$ , the prediction for  $\sigma_{\chi p}$  still varies by almost three orders of magnitude within the range of models we studied; in contrast, the prediction for  $\Delta a_\mu$  “only” varies by about one order of magnitude; see Fig. 3. One reason for this difference is that  $\sigma_{\chi p}$  measures squared Feynman amplitudes, while  $a_\mu$  is directly proportional to a (loop) amplitude. In addition, we saw that  $\Delta a_\mu$  does not depend on  $\mu$  very sensitively, since various effects tend to cancel. In contrast, the sensitivity of  $\sigma_{\chi p}$  to  $\mu$  is often enhanced by cancellations between diagrams with different  $\mu$ -dependence. A similar remark holds for the  $\tan\beta$  dependence. Fig. 1 shows that it is “only” linear for  $\Delta a_\mu$ , but much stronger for  $\sigma_{\chi p}$ . In this case the strong sensitivity of  $\sigma_{\chi p}$  is partly due to the dependence of the mass of the heavy CP-even Higgs boson on  $\tan\beta$ , which is a direct consequence of radiative gauge symmetry breaking. Given the theoretical uncertainties involved in predicting Dark Matter detection rates, this strong parameter dependence of  $\sigma_{\chi p}$  is in some sense fortunate; it means that even an order-of-magnitude determination of  $\sigma_{\chi p}$  will allow us to make significant statements about supersymmetric parameters. Given its comparatively mild parameter dependence, it is also fortunate that the prediction of  $a_\mu$  is much cleaner, if we assume that the hadronic uncertainty can be reduced to the level of the expected experimental uncertainty, which seems feasible.

Our overall conclusion is that measurements of  $a_\mu$  and/or  $\sigma_{\chi p}$  might well yield a positive signal in the next few years. These measurements can help to distinguish between different supersymmetric models. Since well motivated models exist where one of these quantities is sizable while the other is small, it is important to measure, or at least constrain, *both* of them as precisely as possible. On the other hand, we also saw that two models with very different scalar spectrum (the  $SO(10)$  model and mSUGRA with  $m^2 \gg M^2$ ) can lead to very similar predictions for both  $a_\mu$  and  $\sigma_{\chi p}$ . This once again illustrates the truism that indirect measurements can provide crucial information, but they can never completely replace direct searches for new particles.

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## References

- [1] For a review, see H. E. Haber and G. L. Kane, Phys. Rep. **117**, 75 (1985).
- [2] E. Witten, Nucl. Phys. B**188**, 513 (1981).
- [3] J. Ellis, S. Kelley and D.V. Nanopoulos, Phys. Lett. B**249**, 441 (1990), and Phys. Lett. B**260**, 131 (1991); C. Giunti, C.W. Kim and U.W. Lee, Mod. Phys. Lett. A**6**, 1745 (1991); U. Amaldi, W. de Boer and H. Fürstenau, Phys. Lett. B**260**, 447 (1991); P. Langacker and M. Luo, Phys. Rev. D**44**, 817 (1991).
- [4] See e.g. ATLAS Collab., ATLAS Detector and Physics Performance Technical Design Report. CERN/LHCC99-19.  
<http://atlasinfo.cern.ch/Atlas/GROUPS/PHYSICS/TDR/access.html>; Proceedings of the “1996 DPF/DPB Summer Study on High-Energy Physics”. Ed. D. G. Cassel,

- L. T. Gennari and R. H. Siemann; E. Accomando *et al.*, [ECFA/DESY LC Physics Working Group Collaboration], Phys. Rep. **299**, (1998), hep-ph/9705442.
- [5] For a review, see G. Jungman, M. Kamionkowski and K. Griest, Phys. Rep. **267**, 195 (1996).
  - [6] DAMA Collaboration, Phys. Lett. B**480**, 23 (2000).
  - [7] CDMS Collaboration, Nucl. Instrum. Meth. A**444**, 345 (2000), astro-ph/0002471.
  - [8] R. Schnee for the CDMS collab., talk at *DARK 2000*, Heidelberg, Germany, July 2000.
  - [9] CRESST Collaboration, Astropart. Phys. **12**, 107 (1999), hep-ex/9904005.
  - [10] R. Carey, plenary talk given at ICHEP2000; see <http://ichep2000.hep.sci.osaka-u.ac.jp/scan/0801/pl/carey/index.html>
  - [11] E821 Collaboration, Nucl. Phys. Proc. Suppl. **76**, 253 (1999)
  - [12] V.W. Hughes and T. Kinoshita, Rev. Mod. Phys. **71**, D133 (1999).
  - [13] The supersymmetric contribution to  $a_\mu$  has previously been studied by numerous authors. A selection includes: J. Ellis, J. Hagelin and D.V. Nanopoulos, Phys. Lett. **116B**, 283 (1982); J.A. Grifols and A. Mendez, Phys. Rev. **D26**, 1809 (1982); R. Barbieri and L. Maiani, Phys. Lett. **B117**, 203 (1982); D.A. Kosower, L.M. Krauss and N. Sakai, Phys. Lett. **B133**, 305 (1983); T.C. Yuan, R. Arnowitt, A.H. Chamseddine and Pran Nath, Z. Phys. **C26**, 407 (1984); H. König, Mod. Phys. Lett. **A7**, 279 (1992); U. Chattopadhyay and Pran Nath, Phys. Rev. **D53**, 1648 (1996), hep-ph/9507386; M. Carena, G.F. Giudice and C.E.M. Wagner, Phys. Lett. **B390**, 234 (1997), hep-ph/9610233.
  - [14] M. Drees and M. M. Nojiri, Phys. Rev. **D47**, 376 (1993).
  - [15] M. Drees and M.M. Nojiri, Phys. Rev. **D48**, 3483 (1993), hep-ph/9307208.
  - [16] T. Moroi, Phys. Rev. **D53**, 6565 (1996); Erratum-ibid. **D56**, 4424 (1997), hep-ph/9512396.
  - [17] Early analyses of neutralino–nucleon scattering include: M.W. Goodman and E. Witten, Phys. Rev. **D31**, 3059 (1985); K. Griest, Phys. Rev. **D38**, 2357 (1988), Erratum-ibid. **D39**, 3802, (1989); M. Srednicki and R. Watkins, Phys. Lett. **B225**, 140 (1989); M. Kamionkowski, Phys. Rev. **D44** 3021 (1991); G.B. Gelmini, P. Gondolo and E. Roulet, Nucl. Phys. **B351**, 623 (1991); J. Ellis and R.A. Flores, Phys. Lett. **B263**, 259 (1991), and Phys. Lett. **B300**, 175 (1993).
  - [18] Recent analyses of neutralino–nucleon scattering in the MSSM include: H. Baer and M. Brhlik, Phys. Rev. **D57**, 567 (1998), hep-ph/9706509; E. Accomando, R. Arnowitt, B. Dutta and Y. Santoso, Nucl. Phys. **B585**, 124 (2000), hep-ph/0001019; J. Ellis, A. Ferstl and K.A. Olive, hep-ph/0007113, and Phys. Lett. **B481**, 304, (2000), hep-ph/0001005; V. Mandic, A. Pierce, P. Gondolo and H. Murayama, hep-ph/0008022; A.B. Lahanas, D.V. Nanopoulos and V.C. Spanos, hep-ph/0009065; A. Bottino, F. Donato, N. Fornengo and S. Scopel, hep-ph/0010203.
  - [19] For a review, see e.g. L.E. Ibáñez and G.G. Ross, in *Perspectives on Higgs physics*, G.L. Kane, editor; hep-ph/9204201,

- [20] M. Drees and M.M. Nojiri, Phys. Rev. **D45**, 2482 (1992).
- [21] J.R. Espinosa and R.-J. Zhang, JHEP **0003**, 026 (2000), hep-ph/9912236; M. Carena, H.E. Haber, S. Heinemeyer, W. Hollik, C.E.M. Wagner and G. Weiglein, Nucl. Phys. **B580**, 29 (2000), hep-ph/0001002.
- [22] A. Djouadi and M. Drees, Phys. Lett. **B484**, 183 (2000), hep-ph/0004205.
- [23] C. Boehm, A. Djouadi and M. Drees, Phys. Rev. **D62**, 035012 (2000), hep-ph/9911496. See also J. Ellis, T. Falk, K.A. Olive and M. Srednicki, Astropart. Phys. **13**, 181 (2000), hep-ph/9905481.
- [24] M. Drees and A. Yamada, Phys. Rev. **D53**, 1586 (1996), hep-ph/9508254.
- [25] See e.g. <http://lephiggs.web.cern.ch/LEPHIGGS/> .
- [26] ALEPH collab., R. Barate et al., hep-ex/0011047.
- [27] See e.g. J.R. Primack, talk at the *4th International Symposium on Sources and Detection of Dark Matter in the Universe (DM 2000)*, Marina del Rey, California, February 2000, astro-ph/0007187.
- [28] M. Drees, and M. M. Nojiri, Nucl. Phys. **B369**, 54 (1992).
- [29] J.L. Feng, K.T. Matchev and F. Wilczek, astro-ph/0008115.
- [30] R. Arnowitt and Pran Nath, Phys. Rev. Lett. **69**, 728 (1992).
- [31] T. Falk, Phys. Lett. **B456**, 171 (1999), hep-ph/9902352.
- [32] S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, Nucl. Phys. **B353**, 591 (1991).
- [33] G. Degrandi, P. Gambino and G.F. Giudice, hep-ph/0009337.
- [34] A. Brignole, L. Ibañez and C. Muñoz, Nucl. Phys. **B422**, 125 (1994), Erratum-ibid. **B436**, 747 (1995), hep-ph/9308271; T. Kobayashi, D. Suematsu, K. Yamada and Y. Yamagishi, Phys. Lett. **B348**, 402 (1995), hep-ph/9408322; A. Brignole, L. Ibañez, C. Muñoz and C. Scheich, Z. Phys. **C74**, 157 (1997), hep-ph/9508258.
- [35] Y. Kawamura, T. Kobayashi and J. Kubo, Phys. Lett. **B405**, 64 (1997), hep-ph/9703320.
- [36] M. Drees, Phys. Lett. **B181**, 279 (1986); Y. Kawamura, H. Murayama and M. Yamaguchi, Phys. Rev. **D51**, 1337 (1995), hep-ph/9406245.
- [37] V. S. Kaplunovsky and J. Louis, Phys. Lett. **B306**, 269 (1993), hep-th/9303040.
- [38] H. Baer, M.A. Diaz, J. Ferrandis and X. Tata, Phys. Rev. **D61** 111701 (2000), hep-ph/9907211.
- [39] J.L. Feng, K.T. Matchev and T. Moroi, Phys. Rev. **D61**, 075005 (2000), hep-ph/9909334.
- [40] J.L. Feng, K.T. Matchev and F. Wilczek, Phys. Lett. **B482**, 388 (2000), hep-ph/0004043.